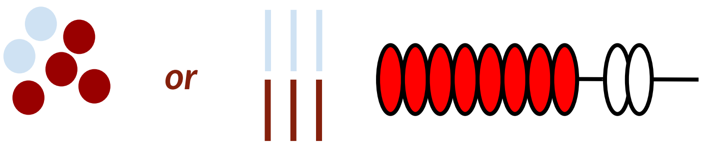
Addition Progression

Reception

Use a variety of manipulatives to begin to understand addition as aggregation (combination of present objects) or augmentation (a change leading to an increase in total):

Make 6   *8 + 2 = 10*

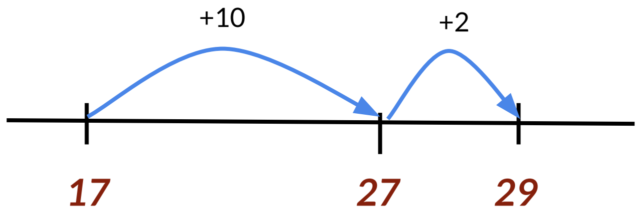


Reception

Year 1

Use a blank number line to add two 2-digit numbers less than 20 by partitioning the smaller addend, then counting on the ten and then the ones:

17 + 12 = 29

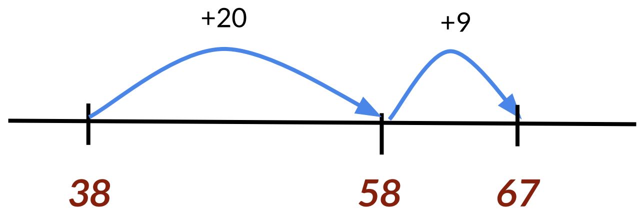


Year 1

Year 2

Use a blank number line to add two 2-digit numbers by partitioning the smaller addend, then counting on the tens and then the ones:

38+ 29 = 67



Year 2

Year 3

Use expanded column addition with up to 3-digit numbers

375 + 167 = 542

300	70	5
+100	60	7
400	130	12
500	40	2

Year 3

Year 4

Use contracted column addition with up to 4-digit numbers and also apply to decimals up to 2d.p. and money:

36.25 + 7.48

$$\begin{array}{r} 36.25 \\ +\ 7.48 \\ \hline 43.73 \\ {\scriptstyle 1\ \ \ 1} \end{array}$$

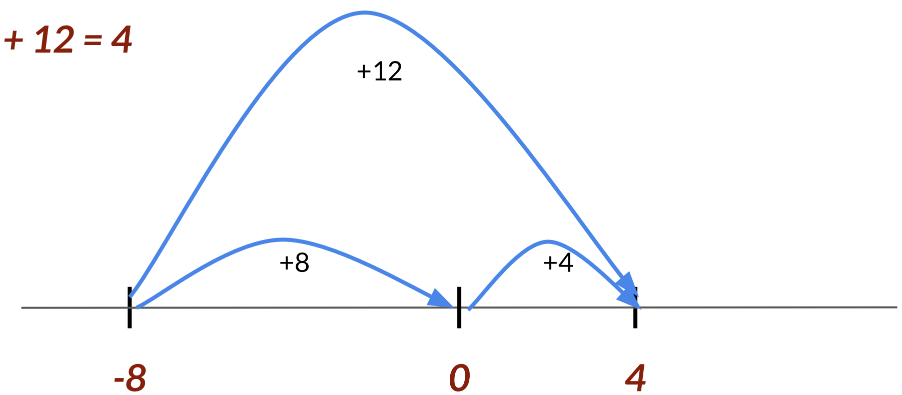
Year 4

Year 5 and 6

Extend contracted written addition with numbers up to 10,000,000 (see year 4 method).

Use a number line to add where the addend is a negative number.

-8 + 12 = 4



Year 5 and 6

Notes on addition and associated mental methods

Reception and year 1 - Children's addition is built on their understanding of perceptual and conceptual subitising with a variety of representations and configurations. Ensure children see representations of ordinal and cardinal aspects of number (i.e. number-line/number-track and dienes/objects). Children also need to be taught to take shortcuts with bead-strings (e.g. counting 12 by recognizing 10 and counting on or subitizing 2 more). Ensure children see representations of ordinal and cardinal aspects of number (i.e. number-line/number-track and dienes/objects). Encourage development towards larger jumps on a number line. Children need to understand that there are different circumstances under which addition is required, i.e. different interpretations of addition: aggregating (grouping like objects), counting on and extending.

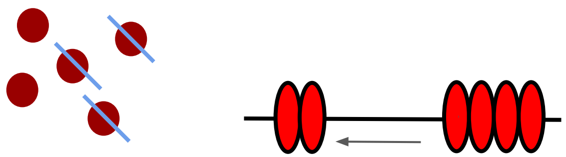
Year 2 - Along with number-lines, dienes and ten-squares should assist with the initial learning of adding tens and ones.

Year 3 - Children should be encouraged to move from explicit use of a number-line to a purely mental method for partitioning a number to bridge through 10 and adding near multiples of ten (compensation). Ensure children use dienes to understand the steps in written addition, including renaming (carrying).

Year 4, 5 and 6 - Further use of compensation will be introduced (e.g. 387 + 574 = 400 + 561 = 961). Children will further develop their mental addition using known facts (e.g. 3 + 9 = 12 so 30 + 90 = 120 so 300 + 900 = 1200, etc); this will also be applied to decimals (e.g 0.3 + 0.9 = 1.2)

Subtraction Progression

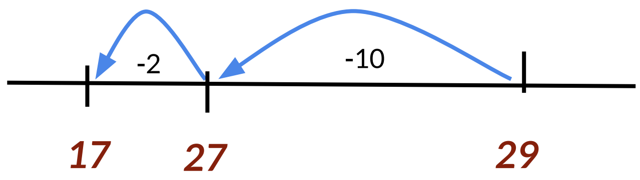
Use a variety of manipulatives to begin to understand subtraction as partitioning (splitting present sets), reduction (specifically, taking away) and finding a difference:



Reception

Use a blank number line to subtract a 2-digit number less than 20 from a 2-digit number less than 30:

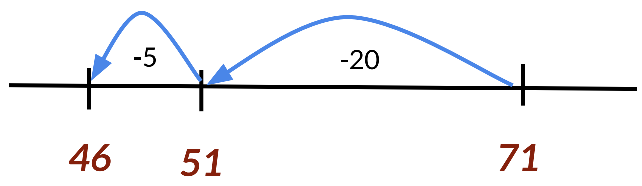
29-12=17



Year 1

Use a blank number line to subtract a 2-digit number from another 2-digit number:

71 - 25 = 46



Year 2

Subtract numbers up to 3 digits using expanded column subtraction:

754 - 136 = 618

700	$^{40}\cancel{5}0$	$^{1}4$
- 100	30	6
600	10	8

Year 3

Use contracted column subtraction with up to 4-digit numbers and applied to decimals up to 2d.p. and money:

26.47 - 2.86 = 3.61

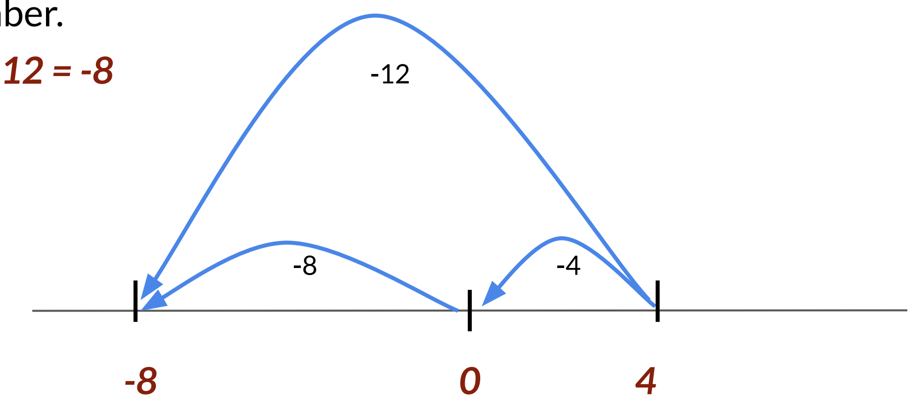
$$\begin{array}{r} 2\,{}^{5}\cancel{6}.{}^{1}4\,7 \\ -\quad 2.8\,6 \\ \hline 2\,3.6\,1 \\ \hline \end{array}$$

Year 4

Extend contracted written subtraction with numbers up to 10,000,000 (see year 4 method).

Use a number line to subtract where the difference is a negative number.

4 - 12 = -8



Year 5 and 6

NB: Written subtraction should be taught with the <u>all</u> necessary exchanging in the minuend completed before subtraction of the subtrahend.

Notes on subtraction and associated mental methods:

Reception and year 1- Children's subtraction is built on their understanding of perceptual and conceptual subitising with a variety of representations and configurations. Ensure children see representations of ordinal and cardinal aspects of number (i.e. number-line/number-track and dienes/objects).Children need to understand that there are different circumstances under which subtraction is required, i.e. different interpretations of subtraction: taking away, counting back, shortening and finding a difference.

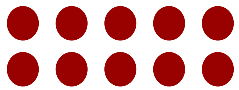
Year 2 - Along with number-lines, dienes and ten-squares should assist with the initial learning of subtracting tens and ones.

Year 3 - Children should be encouraged to move from explicit use of a number-line to a purely mental method for bridging through 10 and subtracting near multiples of ten (compensation); ensure children use dienes to understand the steps in written subtraction, including renaming (carrying).

Year 4, 5 and 6- Further use of compensation will be introduced (e.g. 842 - 689 = 853 - 700 = 153). Children will further develop their mental subtraction using known facts (e.g. 12 - 3 = 9 so 120 - 30 = 90 so 1200 - 300 = 900, etc); this will also be applied to decimals (e.g 1.2 - 0.3 = 0.9)

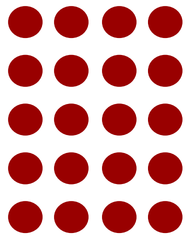
Multiplication Progression

Understand multiplication as repeated addition and visualise this as an array:



= 2 + 2 + 2 + 2 + 2

2 x 5 = 10



= 5 + 5 + 5 + 5

5 x 4 = 20

Year 1 and 2

After understanding the distributive law of multiplication across addition via reference to an area model, use a grid method to multiply 2-digit numbers by 1-digit numbers

x	***20***	***3***
8	160	24

160 + 24 = 184

Year 3

Use contracted multiplication to multiply up to 3-digit numbers by 1-digit numbers:

$$\begin{array}{r} 432 \\ \times \quad 6 \\ \hline 2592 \\ \hline 2\ \ 1\ \ \end{array}$$

Apply the above also to decimals up to 2 decimal places:

$$\begin{array}{r} 4.32 \\ \times \quad 6 \\ \hline 25.92 \\ \hline 2\ \ 1\ \ \ \end{array}$$

Year 4

Use contracted multiplication to multiply up to 4-digit numbers by 2-digit numbers:

$$\begin{array}{rl} 524 & \\ \times\ 37 & \\ \hline 3668 & (524 \times 7) \\ {\scriptstyle 1\ 2}\ \ & \\ 36720 & (524\ \times 30) \\ {\scriptstyle 1\ 1}\ \ \ \ \ \ & \\ \hline 40388 & \end{array}$$

Year 5 and 6

Notes on multiplication and associated mental methods:

Reception and year 1 - Children need to begin to understand that there are different circumstances under which multiplication is required, i.e. different interpretations of multiplication: repeated addition, area, change in the counting unit and scaling/stretching; (the last of these may need benefit from later introduction)

Year 2 - Along with number-lines, dienes and ten-squares should assist with the initial learning of step-counting in 2s, 5s and 10s.

Year 3 - Children's ability to use formal methods of written multiplication is based on their recall of relevant multiplication facts; when teaching methods of multiplication, base these on multiplication facts that the children *can* recall at this stage. Children should initially learn the grid method as a representation of the distributive principle of multiplication (i.e. a x (b + c) = ab + ac) and see this as related to an area application of multiplication; this can be considered as an extension of a mental method with jottings. The expanded multiplication is a formal method to be learned that also relies upon the distributive principle of multiplication. Children should also begin to use known facts to multiply by multiples of 10 (e.g. 3 x 4 = 12 so 3 x 40 = 120)

Year 4 - Children should begin to multiply mentally (with jottings) by using factors (e.g. 16 x 25 → 4 x 4 x 25 → 4 x 100 = 400). They should also begin to use known facts to multiply 1-digit numbers by multiples of 10, 100, 1000 and 0.1 (e.g. 6 x 8 = 48 so 6 x 800 = 4800)

Year 5 and 6 - Children should multiply mentally using partitioning learned in the grid method in year 3, beginning with 2-digit numbers multiplied by 1-digit numbers

Division Progression

Understand division as the sharing or grouping of objects, and solve division calculations relating to the 2, 5 and 10 multiplication facts:

8 ÷ 2 = 4

Year 1 and 2

Understand division as the sharing or grouping of objects or as scaling down, and solve division calculations relating to the 2, 3, 4, 5, 6 and 8 and 10 multiplication facts:

24 ÷ 6 = 4
64 ÷ 8 = 8
36 ÷ 12 = 3

Year 3

Understand division as the sharing or grouping of objects or as scaling down, and solve division calculations relating to the 2, 3, 4, 5, 6 and 8 and 10 multiplication facts:

24 ÷ 6 = 4
64 ÷ 8 = 8
36 ÷ 12 = 3

Year 4

Use short division to divide up to 4-digit numbers by 1-digit numbers, including remainders in some cases. This should be extended to the context of money and decimals:

$$\begin{array}{r|l} & \ \ 5\ \ 9\ r\,3 \\ \hline 6 & \cancel{3}\,{}^{3}5\,{}^{5}7 \end{array}$$

357 ÷ 6 = 59 r3

Year 5

Use repeated subtraction to divide by 2-digit numbers (though short division may still be chosen in some cases):

972 ÷ 36 = 27

$$\begin{array}{rrl} & 2\ 7 & \\ \hline 36\ | & 9\ 7\ 2 & \\ - & \underline{7\ 2\ 0} & (36 \times \mathbf{20}) \\ & {}^{1}\cancel{2}\,{}^{1}5\ 2 & \\ - & \underline{1\ 8\ 0} & (36 \times \mathbf{5}) \\ & 7\ 2 & \\ - & \underline{7\ 2} & (36 \times \mathbf{2}) \\ & 0 & \end{array}$$

Year 6

Notes on division and associated mental methods:

Reception and year 1 - Children need to understand that there are different circumstances under which division is required i.e. different interpretations of division: sharing and grouping. This should be related to visual representations and manipulatives.

Year 2 - Along with number-lines, dienes and ten-squares should assist with the initial learning of step-counting in 2s, 5s and 10s.

Year 3 - Children's ability to divide is based on their recall of relevant multiplication facts; when teaching methods of multiplication, base these on multiplication facts that the children *can* recall at this stage.

Year 4 - When introducing multiplication of multiples of 10 by a single-digit number (e.g. 3 x 50 = 150), the related division facts should also be introduced (e.g. 150 ÷ 3 = 50). Using a constant ratio for division will be introduced (e.g. 450 ÷ 15 = 900 ÷ 30 = 90 ÷ 3 = 30)

Year 5 and 6 - When introducing mental multiplication of decimals by a single-digit number (e.g. 0.3 x 8 = 2.4), the related division facts should also be introduced (e.g. 2.4 ÷ 0.3 = 8). Mental methods that use the law of distributivity of multiplication over division will also be introduced (e.g. 12 x 99 = {12 x 100} - {12 x 1} = 1200 - 12 = 1188)

<u>Learning points underpinning mental addition and subtraction</u>

When comparing two sets of objects, one set can contain more objects than the other and one set can contain fewer objects than the other, or both sets can contain the same number of objects; the symbols <, > and = can be used to express the relative number of objects in two sets, or the relative size of two numbers.

The equals sign in an equation shows equivalence. It does not mean give an answer. This can be demonstrated by showing equations with parts and wholes on different sides of the equation (e.g. 4 + 1 = 5 **and** 5 = 4 + 1)

Parts and whole as a concept can be understood as part of a single object (e.g. apple in two pieces); parts and whole can be represented as parts of a discrete set (e.g. whole: 5 counters; parts: 2 blue counters and 3 red counters

Parts and whole can be split using a cherry diagram; wholes can be split into parts in a variety of ways and the parts always add up to the whole; (this needs to be seen with physical objects in a cherry diagram before working with numbers); the parts and whole can be identified even when one of the parts is zero (i.e. one of the parts is the same value as the whole)

The bringing together of parts to make a whole is one way of visualising addition that relates to given real-life contexts; this is called aggregation. (‘Aggregation’ is not a term with which children need to be familiar.) **Children will need to see this with a lot of different objects to truly grasp what addition means in this context.**

The separation of a whole into parts is one way of visualising subtraction that relates to given real-life contexts; this is called partition. (‘Partition’ is not a term with which children need to be familiar.) **Children will need to see this with a lot of different objects to truly grasp what subtraction means in this context.**

Numbers can represent how many objects there are in a set; for small sets we can recognise the number of objects (subitise) instead of counting them; this can and should be done with different organisations of objects (straight line, dots on a dice, random, in a five/ten frame); numbers to five can be recognised as one set immediately, known as perceptual subitising)

Ordinal numbers indicate a single item or event, rather than a quantity; this is best represented on a number track (or a number-line, though this may be less suitable for the first learning of ordinal numbers)

Learning points underpinning mental addition and subtraction

Each of the numbers one to five can be partitioned in different ways and in can also be partitioned systematically, showing the pattern in parts (e.g. 4 = 4 + 0; 4 = 3 + 1; 4 = 2 + 2; 4 = 1 + 3; 4 = 0 + 4)

Each of the numbers 1-5 can be partitioned into two parts; if we know one part, we can find the other part; this can be represented in a cherry diagram (see above) and a bar model.

The number before a given number is one less; the number after a given number is one more.

The numbers 6-9 can be represented as 5 and a bit (and represented without counting on fingers, bead strings and ten-frames). Quick visual recognition of these numbers through the combination of numbers that are perceptually subitised supports the development of number bonds and is called conceptual subitising.

Each of the numbers 6-9 can be partitioned into two parts; if we know one part, we can find the other part; this can be represented in a cherry diagram (see above) and a bar model.

Even numbers can be visualised in groups of two on a ten-frame, and odd numbers can be visualised in groups of two with one extra; ten-frames and numicon can be used as visualisation tools for this.

The increasing of an amount is one way of visualising addition that relates to given real-life contexts; this is called augmentation. ('Augmentation' is not a term with which children need to be familiar.) This can be shown via a 'First...Then...Now' simple narrative structure. (e.g. First, there were two children on a bus. Then, four children got on the bus. Now there are six children on the bus. → 2 + 4 = 6)

The decreasing of an amount is one way of visualising subtraction that relates to real life contexts; this is called reduction. ('Reduction' is not a term with which children **need** to be familiar, though it may be useful..) This can be shown via a 'First...Then...Now' simple narrative structure. (e.g. First, there were seven children on a bus. Then, three children got off the bus. Now there are four children on the bus. → 7 - 3 = 4)

Addition and subtraction are inverse operations, and subtraction can be seen as the additive inverse of addition (i.e. subtraction can be thought of as what needs to be added to a part to make a total).

Learning points underpinning mental addition and subtraction

Counting on is a way of visualising addition. This can be demonstrated using a number line.

Counting back is a way of visualising subtraction. This can be demonstrated using a number line.

Finding a difference is a way of visualising subtraction. This can be demonstrated using a bar model with two bars of different sizes (known as a comparative bar model).

Addition is commutative: when the order of the addends is changed, the sum remains the same.

The numbers from two to nine can be partitioned into pairs. Recall of these pairs of numbers supports calculation. These can be taught via a variety of strategies including use of ten-frames

Ten can be partitioned into pairs of numbers that sum to ten. Recall of these pairs of numbers supports calculation.

Adding one gives one more; subtracting one gives one less.

Consecutive numbers have a difference of one; we can use this to solve subtraction equations where the subtrahend is one less than the minuend.

Adding two to an odd number gives the next odd number; adding two to an even number gives the next even number. Subtracting two from an odd number gives the previous odd number; subtracting two from an even number gives the previous even number.

Consecutive odd / consecutive even numbers have a difference of two; we can use this to solve subtraction equations where the subtrahend is two less than the minuend.

When zero is added to a number, the number remains unchanged; when zero is subtracted from a number, the number remains unchanged.

Subtracting a number from itself gives a difference of zero.

Doubling a whole number always gives an even number and can be used to add two equal addends; halving is the inverse of doubling and can be used to subtract a number from its double. Doubling and halving can be represented on a ten-frame (or two ten-frames).

Learning points underpinning mental addition and subtraction

Memorised doubles can be used to calculate near-doubles. This takes advantage of the associative property of addition, which states that you can add numbers regardless of how they are grouped, e.g. 8 + 7 → 7 + (7 + 1) → (7 + 7) + 1 ***This shows a mental process and does not necessarily relate to how children will represent calculations on paper.*** One ten is equivalent to ten ones.

We can count in multiples of ten.

Adding ten to a multiple of ten gives the next multiple of ten; subtracting ten from a multiple of ten gives the previous multiple of ten.

Known facts for the numbers *within* ten can be used to add and subtract in multiples of ten by unitising (e.g. 30 + 40 → 3 tens + 4 tens → 7 tens → 70) ***This shows a mental process and does not necessarily relate to how children will represent calculations on paper.***

There is a set counting sequence for counting to 100 and beyond.

The numbers 11–19 can be formed by combining a ten and ones, and can be partitioned into a ten and ones.

Objects can be counted efficiently by making groups of ten. The digits in the numbers 20–99 tell us about their value.

Each number on the 0–100 number line has a unique position. Visualising number lines to 100 and using them is an essential component of understanding number.

The relative size of two two-digit numbers can be determined by first examining the tens digits and then, if necessary, examining the ones digits, with reference to the cardinal or ordinal value of the numbers.

Each two-digit number can be partitioned into a tens part and a ones part. Numbers above ten can often be conceptually subitised if recognisable groups of ten are created.

The tens and ones structure of two-digit numbers can be used to support additive calculation; this can be achieved by partitioning both numbers e.g. 34 + 27 → (30 + 20) + (4 + 7) → 50 + 11 = 61; it can also be achieved by partitioning one number, ideally the smaller one, e.g. 34 + 27 → 34 + (20 + 7) → (34 + 20) + 7 → 54 + 7 = 61 ***This shows a mental process and does not necessarily relate to how children will represent calculations on paper.***

Learning points underpinning mental addition and subtraction

A number is even if the ones digit is even; it *can* be made from groups of two. A number is odd if the ones digit is odd; it *can't* be made from groups of two.

Doubling the numbers 6–9 (inclusive) gives an even teen number; halving an even teen number gives a number from six to nine (inclusive).

Addition and subtraction facts within 10 can be applied to addition and subtraction within 20.

Addition of three addends can be described by an aggregation story with three parts.

Addition of three addends can be described by an augmentation story with a 'First…, Then…, Then…, Now…' structure.

The order in which addends (parts) are added or grouped does not change the sum (associative and commutative laws).

When we are adding three numbers, we choose the most efficient order in which to add them, including identifying two addends that make ten (combining).

We can add two numbers which bridge the tens boundary by using a 'make ten' strategy. This can be visualised using two ten-frames with the transfer of counters or objects. This relies on very secure knowledge of number bonds inside 10 **and** number bonds to 10.

We can subtract across the tens boundary by subtracting *through* ten or subtracting from ten. This relies on very secure knowledge of number bonds inside 10 **and** number bonds to 10.

Difference compares the number of objects in one set with the number of objects in another set; or the difference between two measures.

Difference is one of the structures of subtraction. It can be visualised on a number line or as a bar model.

Learning points underpinning mental addition and subtraction

Known facts for the numbers *within* ten can be applied to addition/subtraction of a single-digit number to/from a two-digit number.

Knowledge of numbers which sum to ten can be applied to the addition of a single-digit number and two-digit number that sum to a multiple of ten, or subtraction of a single-digit number from a multiple of ten.

Known strategies for addition or subtraction bridging ten can be applied to addition or subtraction bridging a multiple of ten.

Learning points underpinning mental multiplication and division

Counting efficiently in groups of 2, 5 and 10 is an essential step towards multiplication.

A coin has a value which is independent of its size. The number of coins in a set is different to the value of coins in a set. The value of a set of identical coins can be calculated using knowledge of counting in groups of 2, 5 and 10.

Objects can be grouped into equal and unequal groups. Equal groups can be represented with a repeated addition expression or a multiplication expression.

Repeated addition of equal groups is one way of visualising multiplication.

Multiplication expressions can be written for cases where the groups each contain zero items or one item.

Changing the counting unit is one way of visualising multiplication.

Increasing by a scale factor (e.g. stretching) is one way of visualising multiplication.

Arrays (and area as an example of this) are one way to visualise multiplication.

Grouping (inverse of multiplication) is one way of visualising division.

Equal sharing is one way visualising division.

Doubling a whole number always gives an even number and can be used to add two equal addends; halving is the inverse of doubling and can be used to subtract a number from its double. Doubling and halving can be represented on a ten-frame (or two ten-frames).

Doubling the numbers 6–9 (inclusive) gives an even teen number; halving an even teen number gives a number from six to nine (inclusive).

The order in which factors are multiplied doesn't affect the product. This called the commutative law of multiplication. Division is not commutative.

Multiplying a number by a group of numbers added together is the same as doing each multiplication separately, e.g. 5 x 12 = (5 x 10) + (5 x 2). This is called the distributive law of multiplication.

When multiplying, it doesn't matter how we group the factors, e.g. (6 x 4) x 3 = 6 x (4 x 3)